Making the most out of noisy quantum computers: Strategies for circuit design and error mitigation

Stefan Kühn

EuroCC
May 2021

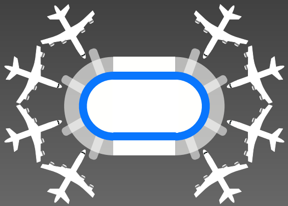
Motivation

Problems for which quantum computers might be advantageous

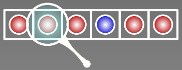
- Factoring

  $$70747 = 263 \times 269$$

- Optimization

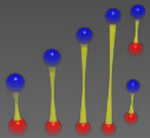


- Searching databases



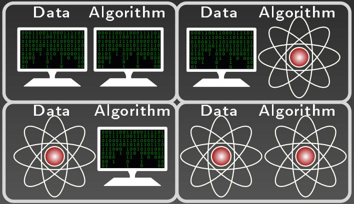
- Quantum simulation
  - Quantum chemistry

  - Particle physics

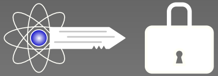


  - Cosmology
  - Material science
  - ...

- Machine learning



- Cryptography

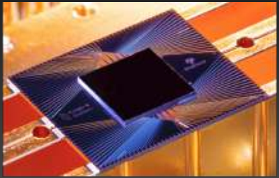


- ...

Motivation

On the verge of the NISQ era

- Noisy Intermediate-Scale Quantum (NISQ) technology (50-100 qubits) is already available
- First small/scale devices are commercially available
- Noise significantly limits the circuit depths that can be executed reliably
  - Not fault tolerant
  - No quantum error correction



J. Preskill, Quantum 2, 79 (2018)

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Article

Quantum supremacy using a programmable superconducting processor

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F. Arute et al., Nature 574, 5050 (2019)

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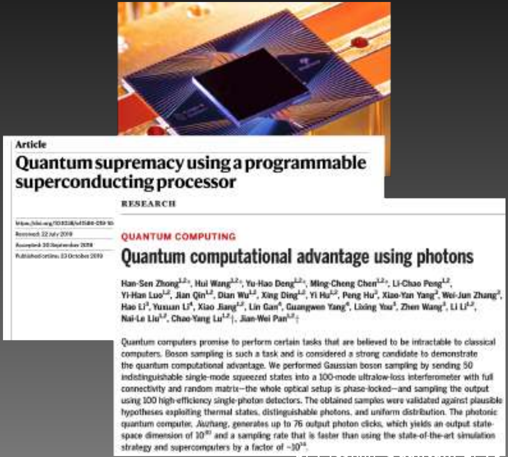

Article

Quantum supremacy using a programmable superconducting processor

RESEARCH

QUANTUM COMPUTING

Quantum computational advantage using photons

Han-Sen Zhong[1,2]*, Hui Wang[1,2]*, Yu-Hao Deng[1,2]*, Ming-Cheng Chen[1,2]*, Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2]†, Jian-Wei Pan[1,2]†

Quantum computers promise to perform certain tasks that are believed to be intractable to classical computers. Boson sampling is such a task and is considered a strong candidate to demonstrate the quantum computational advantage. We performed Gaussian boson sampling by sending 50 indistinguishable single-mode squeezed states into a 100-mode ultralow-loss interferometer with full connectivity and random matrix—the whole optical setup is phase-locked—and sampling the output using 100 high-efficiency single-photon detectors. The obtained samples were validated against plausible hypotheses exploiting thermal states, distinguishable photons, and uniform distribution. The photonic quantum computer, *Jiuzhang*, generates up to 76 output photon clicks, which yields an output state-space dimension of 10^{30} and a sampling rate that is faster than using the state-of-the-art simulation strategy and supercomputers by a factor of ~10^{14}.

(2018)
F. Arute et al., Nature 574, 5050 (2019)
H.-S. Zhong et al., Science 370, 1460 (2020)

Outline

1. Motivation
2. The Circuit Model of Quantum Computing
3. Dimensional Expressivity Analysis
4. Measurement Error Mitigation
5. Summary & Outlook

The Circuit Model of Quantum Computing

Quantum bit

- Qubit: two-dimensional quantum system
- Hilbert space $\mathcal{H}$ with basis $\{|0\rangle, |1\rangle\}$, called the **computational basis**
- Contrary to classical bits, it can be in a **superposition**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad \alpha, \beta \in \mathbb{C}, \qquad |\alpha|^2 + |\beta|^2 = 1$$

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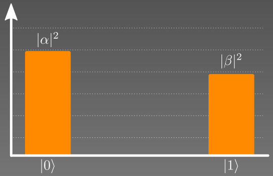
- Measuring the qubit in the computational basis collapses the state onto one of the basis states
  - Probabilities of measuring the two outcomes 0 and 1

$$p(0) = |\alpha|^2, \qquad |\psi\rangle = |0\rangle$$
$$p(1) = |\beta|^2, \qquad |\psi\rangle = |1\rangle$$



- Unlike for classical systems **measurement changes the quantum system**

The Circuit Model of Quantum Computing

Multiple quantum bits

- N qubits: Hilbert space is the tensor product $\underbrace{\mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{N \text{ times}}$
- Most general state in the computational basis

$$|\psi\rangle = \sum_{i_1,\ldots,i_N=0}^{1} c_{i_1\ldots i_N} |i_1\rangle \otimes \cdots \otimes |i_N\rangle$$

- Multiple qubits can be **entangled**

The Circuit Model of Quantum Computing

Entanglement of bipartite systems

- Consider bipartite systems $\mathcal{H}_A \otimes \mathcal{H}_B$
- A quantum state that can be factored as a tensor product of states of its local constituents is called **separable state** or **product state**

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

- Otherwise the state is **entangled**

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Example

- $|\psi_1\rangle = \frac{1}{2}\left(|0\rangle \otimes |0\rangle + |0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right)$

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  $= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
  $\Rightarrow$ product state

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- $|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right)$
  $\Rightarrow$ entangled state (Bell state)

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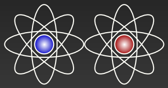
Entanglement of bipartite systems

- Let us consider the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$



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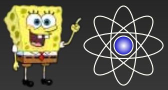

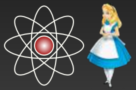

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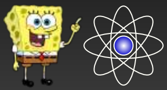

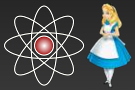

- Bob can measure his qubit

$$p_{\mathrm{Bob}}(0) = \frac{1}{2},$$
$$p_{\mathrm{Bob}}(1) = \frac{1}{2},$$

$\Rightarrow$ Bob does not obtain information about the state

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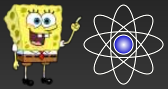

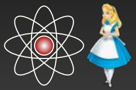

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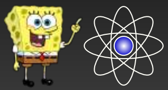

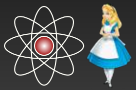

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$$p_{\text{Bob}}(0) = \frac{1}{2}, \qquad |\psi\rangle = |0\rangle \otimes |0\rangle\,, \qquad p_{\text{Alice}}(0) = 1$$

$$p_{\text{Bob}}(1) = \frac{1}{2}, \qquad |\psi\rangle = |1\rangle \otimes |1\rangle\,, \qquad p_{\text{Alice}}(1) = 1$$

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- If Alice measures after Bob she obtains the same result with certainty

$\Rightarrow$ The measurement outcomes are perfectly correlated

The Circuit Model of Quantum Computing

Entanglement of bipartite systems

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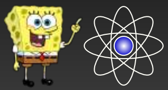

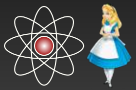

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- If Alice measures after Bob she obtains the same result with certainty
  $\Rightarrow$ The measurement outcomes are perfectly correlated
- Choice of measurement at one location affects the other qubit
  $\Rightarrow$ "Spooky action at a distance"

The Circuit Model of Quantum Computing

Quantum gates

- Quantum mechanics is reversible, $|\psi\rangle$ undergoes unitary evolution under some (time-dependent) Hamiltonian $\mathcal{H}(t)$

$$|\psi(t)\rangle = \underbrace{T\exp\left(-i\int_0^t ds\,\mathcal{H}(s)\right)}_{\text{unitary matrix } U}|\psi_0\rangle$$

- **Quantum gates** are represented by **unitary matrices**

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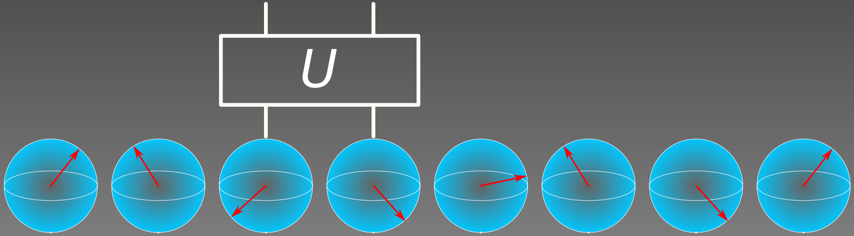
- **Quantum gates** are represented by **unitary matrices**
- Typically gates only act on a few qubits in a nontrivial way



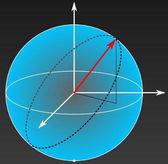
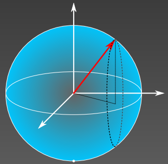
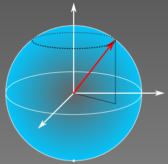
The Circuit Model of Quantum Computing

Common single-qubit quantum gates

Hadamard	$-\boxed{H}-$	$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\lvert 0\rangle \to \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$ $\lvert 1\rangle \to \frac{1}{\sqrt{2}}(\lvert 0\rangle - \lvert 1\rangle)$
X	$-\boxed{X}-$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\lvert 0\rangle \to \lvert 1\rangle$ $\lvert 1\rangle \to \lvert 0\rangle$
Y	$-\boxed{Y}-$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\lvert 0\rangle \to -i\lvert 1\rangle$ $\lvert 1\rangle \to i\lvert 0\rangle$
Z	$-\boxed{Z}-$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\lvert 0\rangle \to \lvert 0\rangle$ $\lvert 1\rangle \to -\lvert 1\rangle$

The Circuit Model of Quantum Computing

Common single-qubit rotations

$R_X(\theta)$	$—\boxed{R_X(\theta)}—$	$R_X(\theta) = \exp\left(-i\frac{\theta}{2}X\right)$	
$R_Y(\theta)$	$—\boxed{R_Y(\theta)}—$	$R_Y(\theta) = \exp\left(-i\frac{\theta}{2}Y\right)$	
$R_Z(\theta)$	$—\boxed{R_Z(\theta)}—$	$R_Z(\theta) = \exp\left(-i\frac{\theta}{2}Z\right)$	

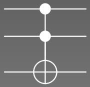
The Circuit Model of Quantum Computing

Common multi-qubit quantum gates

CNOT 	$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert 0\rangle \otimes \lvert 0\rangle \to \lvert 0\rangle \otimes \lvert 0\rangle$ $\lvert 0\rangle \otimes \lvert 1\rangle \to \lvert 0\rangle \otimes \lvert 1\rangle$ $\lvert 1\rangle \otimes \lvert 0\rangle \to \lvert 1\rangle \otimes \lvert 1\rangle$ $\lvert 1\rangle \otimes \lvert 1\rangle \to \lvert 1\rangle \otimes \lvert 0\rangle$

The Circuit Model of Quantum Computing

Common multi-qubit quantum gates

CNOT		$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\|0\rangle \otimes \|0\rangle \to \|0\rangle \otimes \|0\rangle$ $\|0\rangle \otimes \|1\rangle \to \|0\rangle \otimes \|1\rangle$ $\|1\rangle \otimes \|0\rangle \to \|1\rangle \otimes \|1\rangle$ $\|1\rangle \otimes \|1\rangle \to \|1\rangle \otimes \|0\rangle$
SWAP gate		$\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\|0\rangle \otimes \|0\rangle \to \|0\rangle \otimes \|0\rangle$ $\|0\rangle \otimes \|1\rangle \to \|1\rangle \otimes \|0\rangle$ $\|1\rangle \otimes \|0\rangle \to \|0\rangle \otimes \|1\rangle$ $\|1\rangle \otimes \|1\rangle \to \|1\rangle \otimes \|1\rangle$
Toffoli gate		$\text{Toffoli} = \begin{pmatrix} \mathbb{1}_{6\times 6} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	CCNOT

The Circuit Model of Quantum Computing

Quantum gates

- The reversible classical gates can be implemented on a quantum computer
  $\Rightarrow$ We can replicate classical computation

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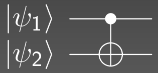
- The Hadamard gate can **create superpositions** out of a single basis state

$$|0\rangle \to |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- The CNOT gate can **create entanglement**



$$|\psi_1\rangle \otimes |\psi_2\rangle \to |\phi_{12}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$$

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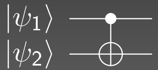


$$|\psi_1\rangle \otimes |\psi_2\rangle \rightarrow |\phi_{12}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$$

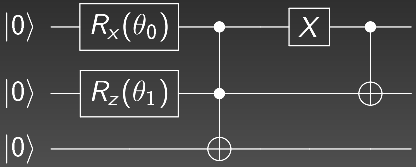
- Since quantum mechanics is linear, we can **apply gates to superpositions of basis states**

$$\begin{aligned}\mathrm{CNOT}\big(&\alpha|0\rangle \otimes |0\rangle + \beta|0\rangle \otimes |1\rangle + \gamma|1\rangle \otimes |0\rangle + \delta|1\rangle \otimes |1\rangle\big) \\ =&\alpha|0\rangle \otimes |0\rangle + \beta|0\rangle \otimes |1\rangle + \gamma|1\rangle \otimes |1\rangle + \delta|1\rangle \otimes |0\rangle\end{aligned}$$

The Circuit Model of Quantum Computing

Quantum circuits

- Combining multiple gates we can build **quantum circuits**



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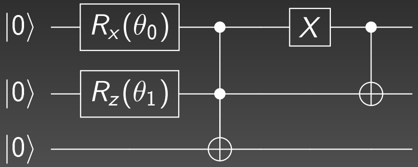

- **Depth** of a circuit: maximum length of a directed path from the input to the output

The Circuit Model of Quantum Computing

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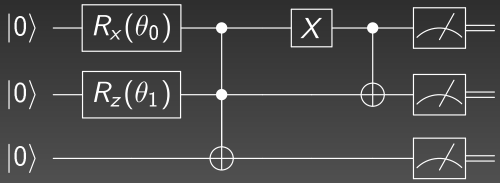
- Combining multiple gates we can build **quantum circuits**



- **Depth** of a circuit: maximum length of a directed path from the input to the output
- Extracting information: final measurement of the qubits (usually in the computational basis)

The Circuit Model of Quantum Computing

Why is Quantum Computing more powerful?

- The Hilbert space of N qubits is the tensor product $\underbrace{\mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{N \text{ times}}$

  $\Rightarrow$ Dimension 2^N, **number of basis states grows exponentially**

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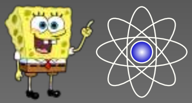
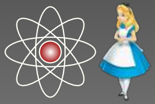
- Multiple **qubits can be entangled**



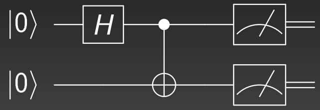


  $\Rightarrow$ **Correlations** that have no classical analog

The Circuit Model of Quantum Computing

Example

- Simple circuit preparing an entangled state (Bell state)



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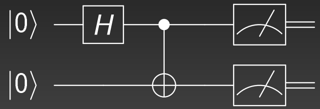

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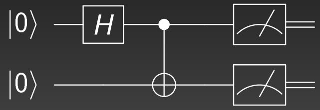

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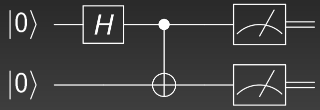

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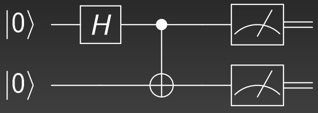

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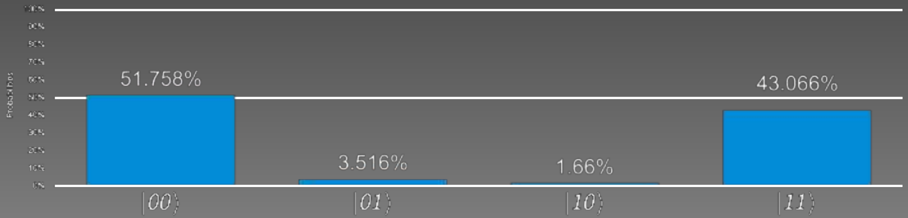
- Results on actual quantum hardware (ibmq_vigo)



The Circuit Model of Quantum Computing

Where do we stand?

Current NISQ devices

- Small or intermediate scale
- Considerable amount of noise
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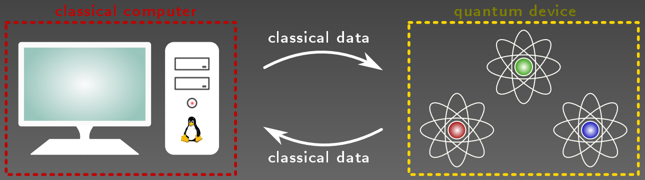
How can we utilize existing quantum hardware in a beneficial way?

The Circuit Model of Quantum Computing

Hybrid quantum-classical algorithms

- Combine classical and quantum devices
- Rely on classical computing where possible
- Use the quantum device as a coprocessor
  - Tackle the classically hard/intractable part of the problem
  - Feed the classical data obtained from a measurement back to the classical computer



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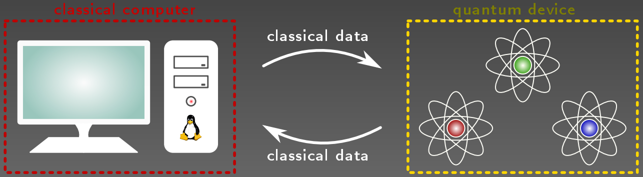

Even modest quantum hardware can yield advantages

The Circuit Model of Quantum Computing

Variational Quantum Eigensolver

- Algorithm to find ground states of quantum Hamiltonians $\mathcal{H}$
- Define a cost function $\mathcal{C}(\vec{\theta}) = \langle\psi(\vec{\theta})|\mathcal{H}|\psi(\vec{\theta})\rangle$, $\vec{\theta} = \mathbb{R}^n$

Peruzzo et al., Nat. Commun. 5, 1 (2014)
J. R. McClean et al., New J. Phys. 18, 023023 (2016)

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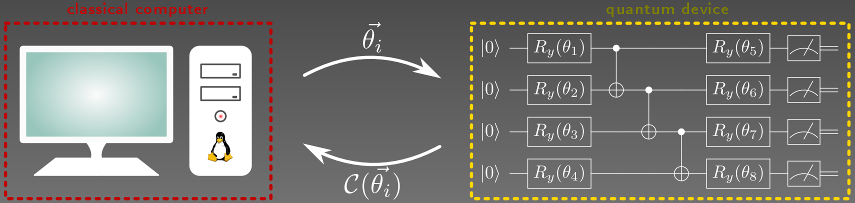

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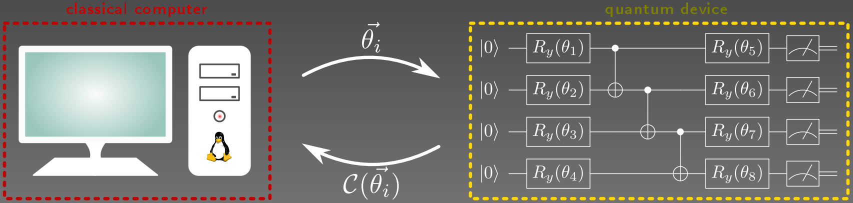

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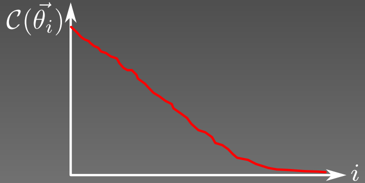

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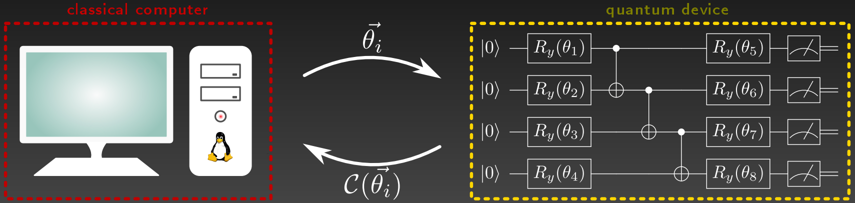
The Circuit Model of Quantum Computing

Variational Quantum Algorithms



Advantages

- Flexible ansatz design
- Hamiltonian exists only as a measurement
- Partially resilient to systematic errors

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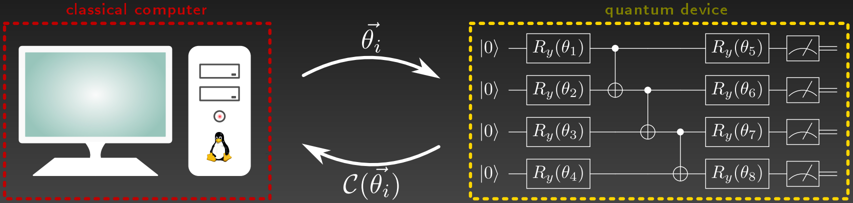

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Challenges

- How to choose an expressive ansatz?
- How to avoid redundant parameters?
- How to deal with effects of noise?

3.

Dimensional Expressivity Analysis

Number of parameters in the ansatz circuit should be

- **large** for solutions to be reachable
- **large** in order not to introduce artificial local optima
- **small** to reduce noise
- **small** for efficient use of many classical optimizers

Dimensional Expressivity Analysis

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Optimal circuit for VQE

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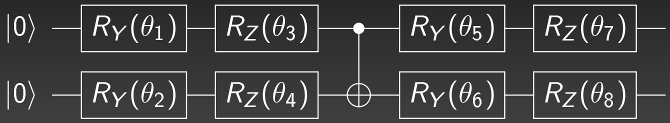
Can we develop a mathematical framework to determine if a circuit is both minimal and maximally expressive?

Dimensional Expressivity Analysis

Dimensional Expressivity Analysis

- Parametric quantum circuit with parameters $\vec{\theta} \in P \subseteq \mathbb{R}^n$ generating $|C(\vec{\theta})\rangle$



- Treat the parametric circuit as a map that maps the input parameters to the state space of the quantum device

$$C : \vec{\theta} \mapsto |C(\vec{\theta})\rangle = R_Z(\theta_8) \dots R_Y(\theta_1) |0\rangle \otimes |0\rangle$$

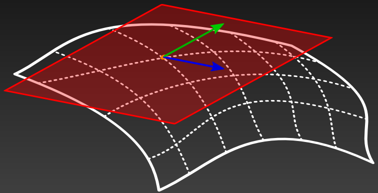
- Parameter space P: real manifold
- Image of C: **circuit manifold** $\mathcal{M}$
- Which parameters are necessary to generate the circuit manifold $\mathcal{M}$?

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Quantum 5, 422 (2021)

Dimensional Expressivity Analysis

Dimensional Expressivity Analysis

- The tangent space of $\mathcal{M}$ is spanned by the tangent vectors $|\partial_j C(\vec{\theta})\rangle$



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Dimensional Expressivity Analysis

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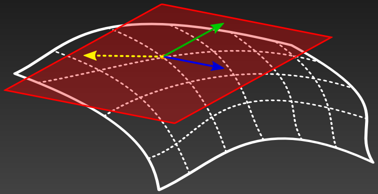
- The tangent space of $\mathcal{M}$ is spanned by the tangent vectors $|\partial_j C(\vec{\theta})\rangle$
- θ_k is redundant iff $|\partial_k C(\vec{\theta})\rangle$ is a linear combination of $|\partial_j C(\vec{\theta})\rangle$, $j \neq k$



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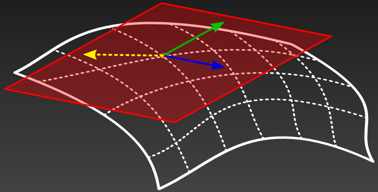

Iterative procedure to identify redundant parameters

- θ_1 is never redundant as long as the corresponding parametric gate is nontrivial
- Check whether $|\partial_{k+1} C(\vec{\theta})\rangle$ is a linear combination of $|\partial_1 C(\vec{\theta})\rangle, \ldots, |\partial_k C(\vec{\theta})\rangle$
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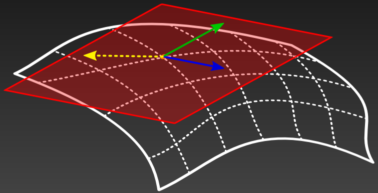

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- Remove redundant parameters
  - Parameter removal implies setting the parameter to a constant value
  - Rotation gates (e.g. $\exp(-\frac{i}{2}\vartheta X)$): choose the parameter $\vartheta = 0$ to achieve an $\mathbb{1}$

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Dimensional Expressivity Analysis

Checking for parameter independence

- θ_1 is never redundant as long as corresponding parametric gate is nontrivial
- For θ_k, $k = 2, \ldots, n$ repeat the following steps
  - Since P is a real manifold, we have to consider the real Jacobian

$$J_k = \begin{pmatrix} | & & | \\ \Re|\partial_1 C\rangle & \ldots & \Re|\partial_k C\rangle \\ | & & | \\ & & \\ | & & | \\ \Im|\partial_1 C\rangle & \ldots & \Im|\partial_k C\rangle \\ | & & | \end{pmatrix}$$

  - If the matrix J_k has full rank then θ_k is independent
- Instead of checking the rank of J_k one can also compute the rank of $S_k = J_k^T J_k$

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- **Memory requirements on a classical computer: exponential in the number of qubits** N because J_k has dimensions $2^{N+1} \times k$

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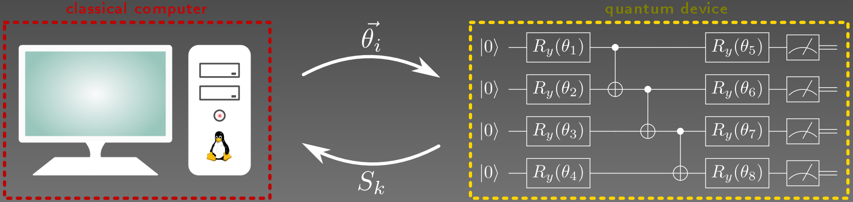
Can we use a hybrid-quantum classical approach for the Dimensonal Expressivity Analysis?



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Dimensional Expressivity Analysis

Hybrid Quantum-Classical Dimensional Expressivity Analysis

- Since the first parameter is always nontrivial $S_1 = \frac{1}{4}$
- For $k \geq 2$ the $k \times k$ matrices $S_k = J_k^T J_k$ can be cast into the form

$$S_k = \begin{pmatrix} S_{k-1} & A_k \\ A_k^T & \frac{1}{4} \end{pmatrix} \qquad \text{with} \qquad A_k = \begin{pmatrix} \Re \left\langle \partial_1 C(\vec{\theta}) \middle| \partial_k C(\vec{\theta}) \right\rangle \\ \vdots \\ \Re \left\langle \partial_{k-1} C(\vec{\theta}) \middle| \partial_k C(\vec{\theta}) \right\rangle \end{pmatrix}$$

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- For $R_G(\vartheta) = \exp(-\frac{i}{2}\vartheta G)$ where G is a gate, the derivative is essentially a circuit

$$|R_G\rangle = |0\rangle \; \text{—}\boxed{R_G(\vartheta)}\text{—} \qquad \Rightarrow \qquad 2i\,|\partial_\theta R_G\rangle = |0\rangle \; \text{—}\boxed{R_G(\vartheta)}\text{—}\boxed{G}\text{—}$$

- Up to an imaginary factor $|\partial_j C(\vec{\theta})\rangle$ can be prepared on a quantum device

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- If we can efficiently obtain $\Re\langle\partial_j C(\vec{\theta})|\partial_k C(\vec{\theta})\rangle$ on the quantum device, we can carry out dimensional expressivity analysis efficiently

S. Lloyd, M. Mohseni, P. Rebentrost, arXiv:1307.0411 (2013)
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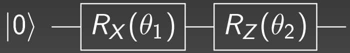

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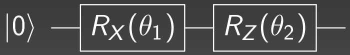
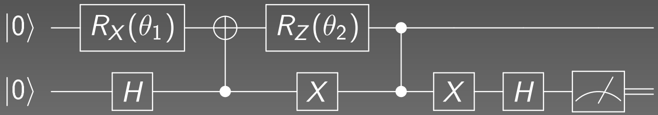


- Circuit for obtaining $\Re\langle \partial_1 C(\vec{\theta}) | \partial_2 C(\vec{\theta}) \rangle$



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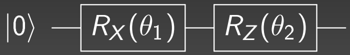
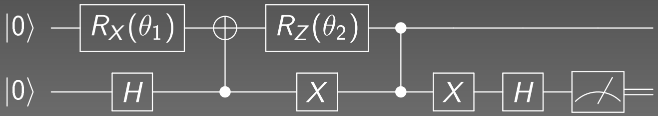


- Circuit for obtaining $\Re\langle\partial_1 C(\vec{\theta})|\partial_2 C(\vec{\theta})\rangle$



- Real part of the overlap is proportional to the probability for the ancilla being in $|0\rangle$

S. Lloyd, M. Mohseni, P. Rebentrost, arXiv:1307.0411 (2013)
L. Zhao, Z. Zhao, P. Rebentrost, J. Fitzsimons, arXiv:1902.10394 (2019)
Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Quantum 5, 422 (2021)

Dimensional Expressivity Analysis

Results for a single qubit on quantum hardware

- Circuit we examine

$$C(\theta_4, \theta_3, \theta_2, \theta_1) = R_Y(\theta_4) R_Z(\theta_3) R_X(\theta_2) R_Z(\theta_1) |0\rangle$$

- Number of independent parameters: 3
- $R_Y(\theta_4)$ is redundant

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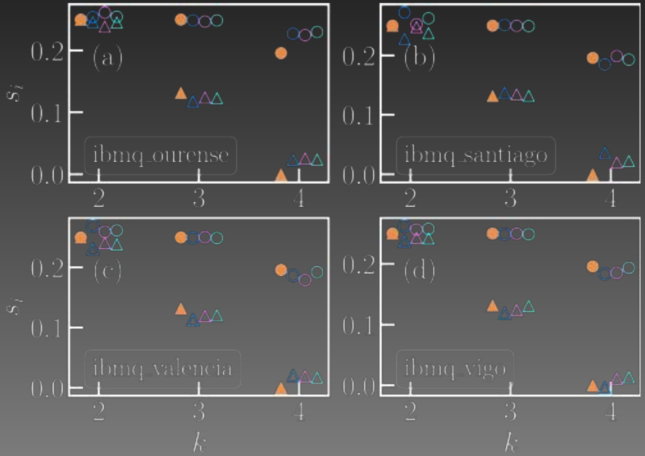
- Results on IBM quantum hardware

- Spectrum of S_k, $k \geq 2$



Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Quantum 5, 422 (2021)

Dimensional Expressivity Analysis

Results for two qubits on quantum hardware

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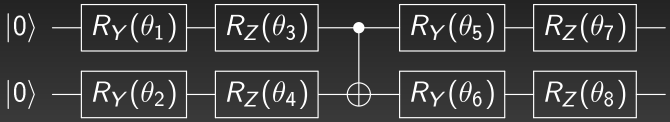
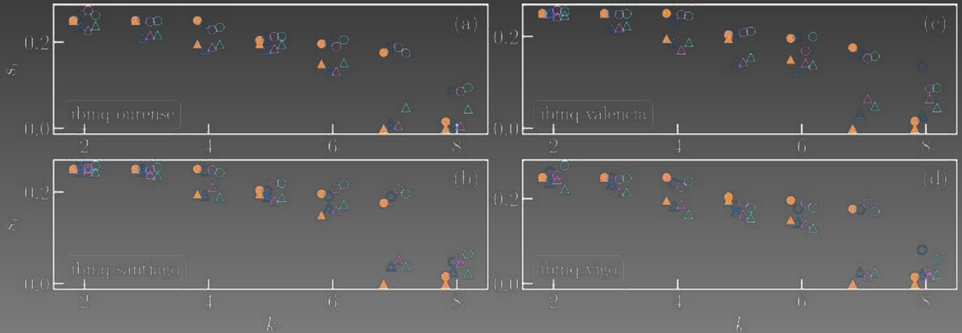




Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Quantum 5, 422 (2021)

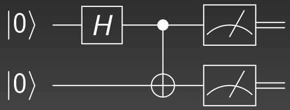
Dimensional Expressivity Analysis

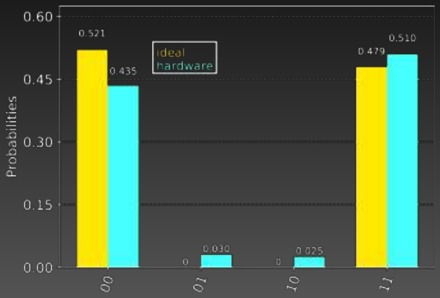
Summary

- Allows for optimizing a given circuit by identifying and **removing redundant parameters**
- Makes it possible to remove unwanted symmetries as well
- Can be **efficiently** performed using a **hybrid quantum-classical** approach

Measurement Error Mitigation

Noise on current quantum devices





Suguru Endo, Zhenyu Cai, Simon C. Benjamin, Xiao Yuan, J. Phys. Soc. Jpn. 90, 032001 (2021)

Measurement Error Mitigation

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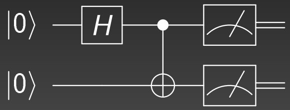

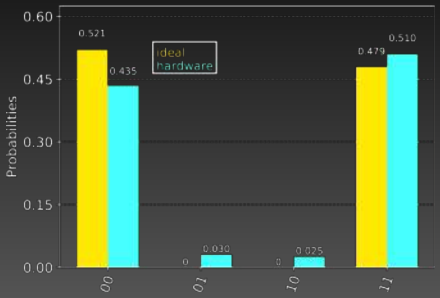

- Errors arise from:
  - Imperfect gates and crosstalk
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  - Measurement/readout

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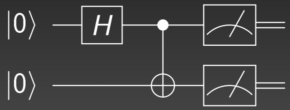

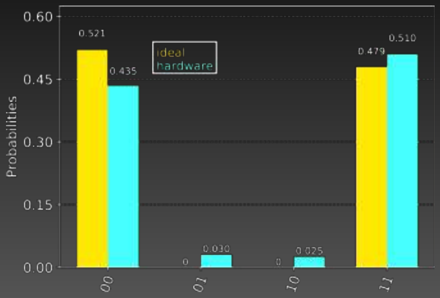

- Errors arise from:
  - Imperfect gates and crosstalk
  - Coupling to environment
  - Measurement/readout
- Typical error rates
  - Single-qubit gates: 0.1% - 0.3%
  - Two-qubit gates: 0.3% - 5%
  - Measurement/readout: 1% - 30%
- **Error mitigation**: Try to correct for (some of) these errors

Suguru Endo, Zhenyu Cai, Simon C. Benjamin, Xiao Yuan, J. Phys. Soc. Jpn. 90, 032001 (2021)

Measurement Error Mitigation

Measurement Error Mitigation

- Focus on a simple **low-overhead**, **resource-efficient** mitigation scheme suitable even for small devices
- Assumptions:
  - **Only measurement errors**, no other sources of noise
  - **Uncorrelated bit flips**, readout errors are not correlated between qubits
  - Bit flips occur with **constant flip probability** for each qubit

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- Assumptions:
  - **Only measurement errors**, no other sources of noise
  - **Uncorrelated bit flips**, readout errors are not correlated between qubits
  - Bit flips occur with **constant flip probability** for each qubit
- Idea: construct random operators $\tilde{O}$ such that such the expectation value subject to noise corresponds to the true expectation value

$$\mathbb{E}\langle\psi|\tilde{O}|\psi\rangle = \langle\psi|O|\psi\rangle$$

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Single-qubit example

- Consider a single qubit with flip probabilities $0 \xrightarrow{p_0} 1$, $1 \xrightarrow{p_1} 0$ and measure the $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ operator

Readout	Bit Flips	Probability	Noisy operator
correct	$0 \to 0,\ 1 \to 1$	$(1 - p_0)(1 - p_1)$	$\tilde{Z} = Z$

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- Expected value of the noisy operator

$$\begin{aligned} \mathbb{E}\tilde{Z} &= (1-p_0)(1-p_1)Z - p_0 p_1 Z - p_0(1-p_1)\mathbb{1} + (1-p_0)p_1\mathbb{1} \\ &= (1-p_0-p_1)Z + (p_0-p_1)\mathbb{1} \end{aligned}$$

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Single-qubit Example

- Expected value of the noisy operator

$$\mathbb{E}\tilde{Z} = (1 - p_0 - p_1)Z + (p_0 - p_1)\mathbb{1}$$

- Reconstruction of the true expectation value

available from calibration on noisy quantum device

$$\langle\psi| Z |\psi\rangle = \frac{1}{1 - p_0 - p_1} \times \mathbb{E}\langle\psi| \tilde{Z} |\psi\rangle - \frac{p_0 - p_1}{1 - p_0 - p_1}$$

true expectation value

measurment on noisy quantum device

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

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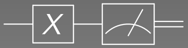
measurment on noisy quantum device

- Calibration of the flip probabilities

$p_{q,0} : |0\rangle$ 

$p_{q,1} : |0\rangle$ 

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Generalization to multiple qubits and arbitrary operators

- General formula for $\tilde{O}_k \in \{\tilde{\mathbb{1}}_k, \tilde{Z}_k\}$

$$\mathbb{E}\left(\tilde{O}_N \otimes \cdots \otimes \tilde{O}_1\right) = \sum_{O \in \{\mathbb{1}, Z\}^{\otimes N}} \Gamma(O_N|\tilde{O}_N) O_N \otimes \cdots \otimes \Gamma(O_1|\tilde{O}_1) O_1$$

where

$$\Gamma(O_q|\tilde{O}_q) = \begin{cases} 1 - p_{q,0} - p_{q,1} & \text{for} \quad \tilde{O}_q = \tilde{Z}_q \wedge O_q = Z_q \\ p_{q,1} - p_{q,0} & \text{for} \quad \tilde{O}_q = \tilde{Z}_q \wedge O_q = \mathbb{1}_q \\ 1 & \text{for} \quad O_q = \mathbb{1}_q \ \wedge \ \tilde{O}_q = \tilde{\mathbb{1}}_q \\ 0 & \text{for} \quad O_q = Z_q \ \wedge \ \tilde{O}_q = \tilde{\mathbb{1}}_q \end{cases}$$

- Set of equations is**invertible** as long as $p_{q,0} + p_{q,1} \neq 1$

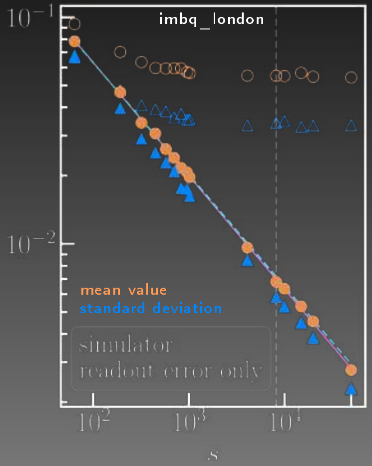
Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Two-qubit case, classical simulation

- Results for a **classical simulation** with **readout errors only**
- Measure the expectation value of $Z \otimes Z$ for 1050 random parameter sets
- Monitor the average and standard deviation of

$$|\,\langle\psi|Z \otimes Z|\psi\rangle_{\text{exact}} - \langle\psi|Z \otimes Z|\psi\rangle_{\text{mitigated}}\,|$$



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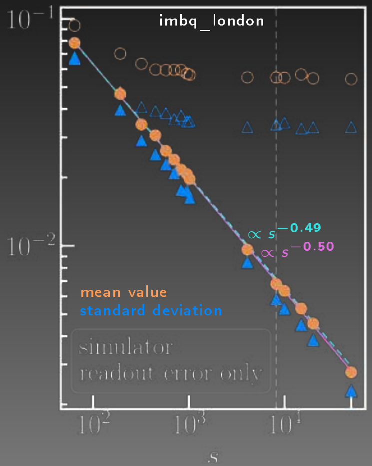


$\Rightarrow$ Mitigated results show power law decay $s^{-1/2}$ just as in the noise-free case

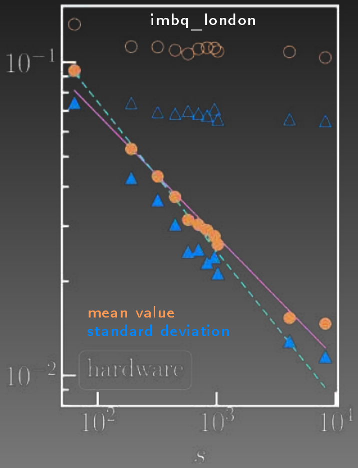
Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Two-qubit case, quantum hardware

- Results for **IBM quantum hardware**
- Measure the expectation value of $Z \otimes Z$ for 1050 random parameter sets
- Monitor the average and standard deviation of

$$\left| \langle\psi|Z \otimes Z|\psi\rangle_{\text{exact}} - \langle\psi|Z \otimes Z|\psi\rangle_{\text{mitigated}} \right|$$



Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

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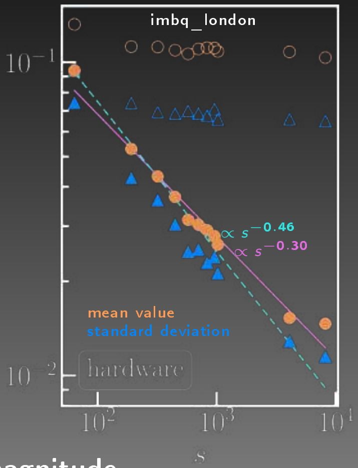

$\Rightarrow$ **Improvement** of the error up to **one order of magnitude**

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Xiaoyang Wang, arXiv:2007.03663 (2020)

Measurement Error Mitigation

Summary

- For **local Hamiltonians** the **overhead cost** is **polynomial**
- It is possible to do a probabilistic version of the mitigation scheme
- The idea of constructing random operators is very general and can potentially be applied to mitigate other kinds of errors
  - Incorporate correlations between qubits
  - Relaxation errors
  - ...

Summary & Outlook

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- Noisy intermediate-scale quantum devices are available
- Hybrid-quantum classical algorithms are promising for these devices
- Dimensional Expressivity Analysis allows for designing minimal maximally expressive circuits for these applications
- Measurement/readout errors can be efficiently mitigated with low overhead

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Outlook

- Generalize Dimensional Expressivity Analysis to be able to quantify the approximation error of a given ansatz
- Extend the mitigation scheme to various other types of error
- Quantum hardware is advancing quickly

Thank you for your attention!

Questions?

Appendix A: Dimensional Expressivity Analysis

Hybrid quantum-classical Dimensional Expressivity Analysis

- $\Re \left\langle \partial_j C(\vec{\theta}) \middle| \partial_k C(\vec{\theta}) \right\rangle$ can be obtained on the quantum device

Appendix A: Dimensional Expressivity Analysis

Hybrid quantum-classical Dimensional Expressivity Analysis

- $\Re \left\langle \partial_j C(\vec{\theta}) \middle| \partial_k C(\vec{\theta}) \right\rangle$ can be obtained on the quantum device
- In general $\Re \langle \psi | \phi \rangle$ can be measured using an ancilla qubit provided one can prepare the state

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes |\phi\rangle \right)$$

- Applying a Hadamard gate on the ancilla one finds

$$(H \otimes \mathbb{1}) |\chi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle \otimes (|\psi\rangle + |\phi\rangle) + |1\rangle \otimes (|\psi\rangle - |\phi\rangle) \right)$$

- Probability of measuring the ancilla in zero

$$p(\text{ancilla} = 0) = \frac{1}{2} \left(1 + \Re \langle \psi | \phi \rangle \right)$$

Lena Funcke, Tobias Hartung, Karl Jansen, SK, Paolo Stornati, Quantum 5, 422 (2021)